

# Multiple motions in the projective plane



INB

Martin Haker, Cicero Mota, and Erhardt Barth  
 {haker,mota,barth}@inb.uni-luebeck.de

University of Lübeck  
 Institute for Neuro- and Bioinformatics  
[www.inb.uni-luebeck.de](http://www.inb.uni-luebeck.de)

## Introduction

Motion selectivity is a key feature of biological visual processing and multiple transparent motions have been widely used to probe vision.

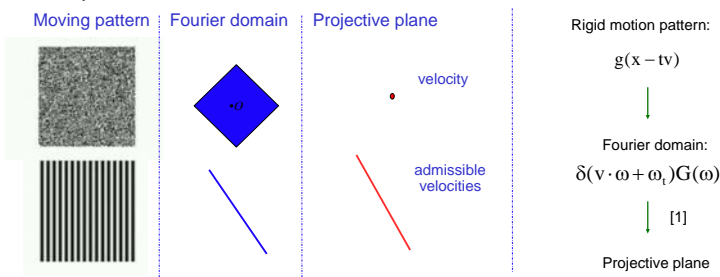
To aid such experimental investigations, we present [1]

- a new theoretical framework for categorizing multiple motions
- an interactive tool that can be used to synthesize multiple overlaid motion patterns in an intuitive and interactive way.

As an extension to classical motion theory we introduce the projective plane as a new way of representing motion patterns.

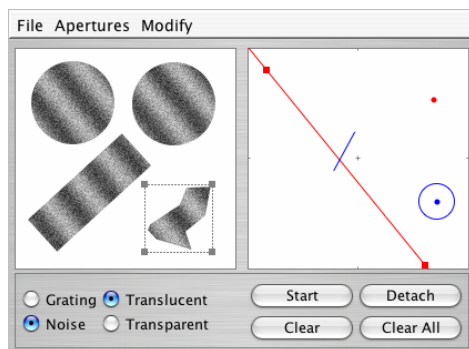
## Motion in the projective plane

The **projective plane (PP)** is the set of all directions in the three-dimensional Euclidean space. The correspondence is such that 2D moving patterns correspond to points and 1D moving patterns correspond to lines in the PP.



## Interactive tool for motion synthesis

The interactive tool lets the user draw points and lines (on the right board) in the PP. The corresponding motions are shown in the left panel. The superposition can be chosen to be transparent or translucent. Single lines in the PP are synthesized as moving gratings. In addition, different apertures can be drawn.



*Small example experiment:* when we draw two lines that intersect, we see the motion of the plaid that corresponds to the intersection of the two lines. When we draw three lines that intersect in one point, we see the motion of the intersection. However, when the three lines intersect in three points, there is no consistent motion percept and this difference, that is perceptually very strong, is elegantly captured by the representation in the projective plane.

The interactive tool will be presented during the poster session. It is available under <http://www.inb.uni-luebeck.de/vision/demos/>

## Multiple motions

Single motion:  $f(x, t) = g(x - tv)$

$$\alpha(v)f = \nabla f^T V = 0 \quad V = (v_x, v_y, 1)^T$$

Multiple motions [1]:

$$f(x, t) = g_1(x - tv_1) + \dots + g_N(x - tv_N)$$

$$\alpha(v_1) \dots \alpha(v_N)f = \sum_I c_I f_I = 0$$

Generalized structure tensor

$$J_N = h^*[(f_I) \otimes (f_I)]$$

convolution kernel

What spatio-temporal patterns correspond to the  $M$  possibilities for the rank of  $J_N$ ?

$$M = (N+1)(N+2)/2$$

Pattern	Intrinsic dimension	Rank $J_1$	Rank $J_2$	Rank $J_3$
0D	0	0	0	0
1D	1	1	1	1
1D+1D	2	2	2	2
1D+1D+1D	3	3	3	3
2D	2	2	3	4
2D+1D	3	3	4	5
2D+1D+1D	3	3	5	6
2D+2D	3	3	5	7
2D+2D+1D	3	3	6	8
2D+2D+2D	3	3	6	9
3D	3	3	6	10

## Conclusions

Our theoretical results provide

- a *categorization of motion* patterns by the rank of the generalized structure tensor
- an intuitive *representation of motion* in the projective plane.

The PP representation has a number of benefits. For example, the motion of two overlaid 1D patterns (e.g. two gratings) can be distinguished from the motion of one 2D pattern. Such patterns remain equivalent within traditional theories of only one motion.

The interactive tool was designed to aid the understanding of more complex motion patterns and the design of visual experiments on motion perception.

## References

[1] <http://www.inb.uni-luebeck.de/locomotor>